

# Stability Analysis for Routh-Hurwitz Conditions Using Partial Pivot

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## Stability Analysis for Routh-Hurwitz Conditions Using Partial Pivot

N Erawaty, Kasbawati and A K Amir

Hasanuddin University, Indonesia

[nurerawaty@gmail.com](mailto:nurerawaty@gmail.com), [kasbawati@gmail.com](mailto:kasbawati@gmail.com), [amirkamalamir@yahoo.com](mailto:amirkamalamir@yahoo.com)

**Abstract.** Stability analysis of the polynomial characteristic equation is not easy to applied to a high-order system, in this case, it relates to search the roots of the degree n polynomial equation. It is difficult to find the roots of the equation, so stability analysis will be carried out using the stability of Routh-Hurwitz. In this study, a polynomial characteristic equation is given with a degree n with a real coefficient, then applied to the Hurwitz Matrix and a Gauss elimination procedure with a partial pivot is performed, then the growth factor is calculated. The results showed that the polynomial characteristic equation of degree n for Routh-Hurwitz conditions is said to be stable if each zero of the polynomial is located in the half left open field if and only if the elimination procedure can be performed and the optimal value of growth factor is 1.

### 1. Introduction

The problem of stability analysis using the characteristic equation is difficult if we applied to a high-order system, in this case, it relates to search the roots of the degree n polynomial equation. With the difficulty of finding the roots of the equation, so that stability analysis will be carried out using the stability of Routh-Hurwitz.

The stability of Routh-Hurwitz relates to the system of polynomial equations, matrices, Gauss elimination with partial pivots and growth factors. Previously discussed in the journal "Algorithms for the Routh-Hurwitz Stability Test" [1] and "Characterization and Stable Test for the Routh-Hurwitz conditions and for total positivity" [2] that if given a polynomial characteristic equation of degree n it can be applied to the Hurwitz matrix and processed by Gauss elimination. However, there are conditions if the elements in the Hurwitz matrix are non-positive, the procedure is stopped just like that. In the Journal of "On The Interlacing of Property and The Routh-Hurwitz Criterion" [3] and "Hermite-Biehler, Routh-Hurwitz" and Total Positivity "[4] discussed the problem of simple proof of Routh-Hurwitz criteria, and Routh-Hurwitz criteria. So in this paper we will discuss how to analyze stability for Routh Hurwitz conditions and determine the optimal growth factor values from the characteristic equation of the polynomial.

The formulation of the problem to be discussed in this journal is how to analyze stability for Routh-Hurwitz conditions and determine the optimal growth factor from the characteristic equation of the polynomial.

We limit the discussion to the characteristic equations of polynomials with degrees n and real value coefficients. We determine growth factors using Gauss elimination with partial pivots based on definitions of existing growth factors. The purpose of writing this journal is to determine the stability of the Routh-Hurwitz condition and the optimal growth factor value from the characteristic equation of the polynomial.

### 2. Stability Criteria for Routh-Hurwitz Conditions

Given a system of characteristic equations in the form of n-order polynomials as follows

$$f(s) = d_0 z^n + d_1 z^{n-1} + \dots + d_{n-1} z + d_n \quad (1)$$

If all the real parts of equation (1) from the root are negative then

$$\frac{d_1}{d_0} > 0, \frac{d_2}{d_0} > 0, \dots, \frac{d_n}{d_0} > 0 \quad (2)$$

[5]

**Definition 1**

Given a polynomial (1) with  $d_k$  real numbers for  $k = 0, 1, 2, \dots, 2n-1$  and  $d_0$  positive numbers. The Hurwitz matrix for equation (1) is defined as a square matrix of size  $n \times n$  as follows:

$$H_n := (d_{2j-1})_{1 \leq i, j \leq n} = \begin{pmatrix} d_1 & d_3 & d_5 & d_7 & \dots & d_{2n-1} \\ d_0 & d_2 & d_4 & d_6 & \dots & d_{2n-2} \\ 0 & d_1 & d_3 & d_5 & \dots & d_{2n-3} \\ 0 & d_0 & d_2 & d_4 & \dots & d_{2n-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & d_n \end{pmatrix} \quad (3)$$

where  $d_k = 0$  for  $k < 0$  or  $k > n$ .

So the matrix element indexed greater than  $n$  or negative index must be replaced by zero.

The  $k$ -level Hurwitz determinant, denoted by  $\det H_k$ ,  $k = 1, 2, \dots, n$  formed from the Hurwitz matrix (3), is defined as follows:

$$\det H_n = \begin{vmatrix} d_1 & d_3 & d_5 & d_7 & \dots & d_{2n-1} \\ d_0 & d_2 & d_4 & d_6 & \dots & d_{2n-2} \\ 0 & d_1 & d_3 & d_5 & \dots & d_{2n-3} \\ 0 & d_0 & d_2 & d_4 & \dots & d_{2n-4} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & d_n \end{vmatrix}$$

**Theorem 1**

The polynomial root (1) has a real part of its root is negative if and only if the inequality (2) is fulfilled and

$$\det H_1 > 0, \det H_2 > 0, \det H_3 > 0, \dots, \det H_n > 0$$

Thus, the equilibrium point  $\bar{z}$  is stable if and only if  $\det H_j > 0$  for each  $j = 1, 2, \dots, n$ . For  $n = 3$  and  $n = 4$ , the criteria for Routh-Hurwitz conditions are given as follows

$n = 3$ ;  $d_1 > 0, d_2 > 0, d_3 > 0, d_1 \cdot d_2 - d_3 > 0$ ,

$n = 4$ ;  $d_1 > 0, d_2 > 0, d_3 > 0, d_4 > 0, d_1 \cdot d_2 - d_3 > 0$ ,

$d_3(d_1 \cdot d_2 - d_3) - d_1^2 \cdot d_4 > 0$  [6]

Given a positive integer  $k, n, 1 \leq k \leq n$ .

Suppose  $Q_{k,n}$  is a submatrix of  $H_n$  whose elements are rows to  $k$  to row to  $n$ . Let  $Q_{l,m}$  be a  $H_n$  submatrix whose elements are columns  $l$  to column  $m$ . Suppose that  $A$  is a block element measuring  $n \times m$  from the matrix with the order  $n$ . For  $k \leq n, l \leq m$  and for all  $\alpha \in Q_{k,n}$  and  $\beta \in Q_{l,m}$ , denoted by  $A[\alpha|\beta]$  is a submatrix of  $A$  that has the size  $k \times l$  where  $\alpha$  contains the location of the number in the row and  $\beta$  contains the location of the number in the column. For short, write  $A[\alpha] := A[\alpha | \alpha]$ .

Routh-Hurwitz conditions are well-known for the character of stable polynomials where polynomials with all real parts of the root are negative.

**Theorem 2.**

Suppose that given a polynomial such as equation (1) which has every zero of the polynomial is located in the half left open field if and only if  $d_k > 0$  for all  $k = 0, 1, \dots, n$  and  $\det H_n[1, \dots, k] > 0$  for all  $k = 1, \dots, n$ . [2].

It is known that if the Routh-Hurwitz condition applies, the Hurwitz matrix is a matrix that is totally positive. Totally positive is a matrix where all the elements are positive.

By considering Theorem 2, the proper stability analysis to examine the condition of Routh-Hurwitz is to determine all pivots that are positively marked after elimination from  $H_n$ . This section will also show the optimal growth factor from the polynomial characteristic equation with degree  $n$ . It can be examined that all the elements obtained from the Routh scheme are  $d_i^{(k)} (1 \leq k \leq n, k \leq i \leq n)$  and

the first column of the Routh scheme is formed by diagonal elements  $d_i^{(i)} (1 \leq i \leq n)$ . So, the basic operation of Gauss elimination will be used with partial pivots and determine the optimal growth factor value to check the condition of Routh-Hurwitz. In addition, all steps to maintain the testing of the Hurwitz matrix structure are related to submatrix  $H_n^{(k)} [k, \dots, n]$ .

**Definition 2.**

The growth factor is defined as  $\rho_n := \frac{\max_{i,k} |d_i^{(k)}|}{\max_i |d_i|}$ , where  $d_i^{(k)}$  is the element of a matrix and  $\rho_n \geq 1$

**3. Gauss Elimination Procedure with partial pivots**

Given the polynomial  $P(z)$  of equation (1) and from Theorem 2, it can be assumed that all coefficients of  $P(z)$  are positive or  $d_k > 0$  for all  $k$ . (If not, then the polynomial is not stable).

Matrix  $H_n^{(1)} := H_n$  and  $d_k^{(1)} = d_k$  for all  $k \in \{0, \dots, n\}$ . Let  $H_n^{(2)}$  is a matrix obtained from  $H_n^{(1)}$  by subtracting every even row  $H_n^{(1)}$  with  $\frac{d_0^{(1)}}{d_1^{(1)}}$  multiplied by the previous row or in other words:

$$b'_{2k} = b_{2k} - \frac{d_0}{d_1} b_{2k-1}$$

then the matrix is obtained

$$H_n^{(2)} = \begin{pmatrix} d_1^{(1)} & d_3^{(1)} & d_5^{(1)} & d_7^{(1)} & \dots & 0 \\ 0 & d_2^{(2)} & d_4^{(2)} & d_6^{(2)} & \dots & 0 \\ 0 & d_1^{(2)} & d_3^{(2)} & d_5^{(2)} & \dots & 0 \\ 0 & 0 & d_2^{(2)} & d_4^{(2)} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & d_n^{(2)} \end{pmatrix} \tag{4}$$

where  $d_{2i+1}^{(2)} = d_{2i+1}^{(1)} = d_{2i+1}$  for all  $i \geq 0$ . If  $d_{2i}^{(2)} \leq 0$  for an  $i \geq 1$ , the elimination procedure is stopped (Only the element in the second row of  $H_n^{(2)}$  should be considered). Then the elimination procedure can proceed.

Suppose  $H_n^{(3)}$  is a matrix obtained from  $H_n^{(2)}$  by subtracting every odd row of  $H_n^{(2)}$  with  $\frac{d_1^{(2)}}{d_2^{(2)}}$  multiplied by the previous row or in other words:

$$b'_{2k+1} = b_{2k+1} - \frac{d_1^{(2)}}{d_2^{(2)}} b_{2k}$$

Then we got the matrix

$$H_n^{(3)} = \begin{pmatrix} d_1^{(1)} & d_3^{(1)} & d_5^{(1)} & d_7^{(1)} & \dots & 0 \\ 0 & d_2^{(2)} & d_4^{(2)} & d_6^{(2)} & \dots & 0 \\ 0 & 0 & d_3^{(3)} & d_5^{(3)} & \dots & 0 \\ 0 & 0 & d_2^{(3)} & d_4^{(3)} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & d_n^{(3)} \end{pmatrix} \tag{5}$$

where  $d_{2i}^{(3)} = d_{2i}^{(2)}$  for all  $i \geq 1$ . If  $d_{2i+1}^{(3)} \leq 0$  for an  $i \geq 1$ , the elimination procedure is stopped (Only the element in the third row of  $H_n^{(3)}$  should be considered). Then you can repeat the previous procedure in the same way. At most we can do  $n-1$  steps from the elimination process to obtain the upper triangular matrix.

$$H_n^{(n)} = \begin{pmatrix} d_1^{(1)} & d_3^{(1)} & d_5^{(1)} & d_7^{(1)} & \dots & 0 \\ 0 & d_2^{(2)} & d_4^{(2)} & d_6^{(2)} & \dots & 0 \\ 0 & 0 & d_3^{(3)} & d_5^{(3)} & \dots & 0 \\ 0 & 0 & 0 & d_4^{(4)} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & d_n^{(n)} \end{pmatrix} \quad (6)$$

as we know that  $d_n^{(n)} = d_n > 0$ .

Suppose it is assumed that  $H_n^{(m)}$  ( $1 \leq m \leq n$ ) is the last matrix obtained before the elimination procedure is stopped. So that elements from  $d_i^{(k)} > 0$ ,  $k \leq m$ , have been obtained before the first non-positive entry appears. From Definition 2, the growth factor of the elimination procedure has been defined as a number

$$\rho := \frac{\max_{i,k} |d_i^{(k)}|}{\max_i |d_i|} \geq 1 \quad (7)$$

by using  $d_i^{(k)} > 0$  which has been calculated before the elimination procedure is stopped. Of course  $\rho \geq 1$  and will be shown in the next results that the elimination procedure carried out has an optimal  $\rho = 1$ .

**Theorem 3.**

Suppose that the polynomial  $P(z)$  is given from the equation (1), then

- (i) The polynomial  $P(z)$  has all its zeros in the half left open field if and only if we can perform all the steps of the previous elimination procedure (so  $d_i^{(k)} > 0$  for all  $1 \leq k \leq n, k \leq i \leq n$ ).
- (ii) The elimination procedure (i) uses Gauss elimination with partial pivots.
- (iii) The optimal growth factor ( $\rho$ ) is 1 or  $\rho > 1$ . [2].

In previous evidence, we have seen that the  $H_n[k, \dots, n]$  matrix obtained from the elimination procedure is also the Hurwitz matrix. So to analyze the stability of the Hurwitz matrix only need to pay attention to the  $d_i^{(k)}$  element and know the number of positive roots and can determine the range of stability values. If it does not determine the positivity of all elements  $d_i^{(k)}$ , then the growth factor can be large arbitrarily, not 1.

**4. Routh-Hurwitz Condition in the Singular Case**

The criteria for Routh-Hurwitz conditions generally only exist in ordinary cases, namely cases where  $P(z)$  has a degree of positive integer coefficient, when applied to equations (3), (5) and (6). However, this process will stop when it is found  $d_k^k = 0$ . This is called the singular case in the Routh-Hurwitz criteria.

Given the equation polynomial  $P(z)$  with degree,

$$P(z) = d_0 z^n + d_1 z^{n-1} + \dots + d_n, d_0 > 0.$$

Suppose that by applying equations (3) and (4), (5) alternately a matrix  $H_n^{(m)}$  is obtained where the leading principal minor of the matrix  $H_n^{(m)}$ ,  $n = 1, 2, \dots, m$  is not zero.

Let

$$P(z) = d_0^m z^n + d_1^m z^{n-1} + \dots + d_n^m$$

where  $d_n^m \neq 0$ , if  $d_{n-1}^m = 0$ , then it is clear that the elimination procedure will be stopped because in the next step the algorithm requires conditions  $d_k^{(k)} > 0$ . So there are cases that might occur as follows.

**Case 1.**

For example there is  $k = 1, 3, 5, \dots$  so that  $d_{n-k}^m \neq 0$ . Let  $l \in \{1, 3, 5, \dots\}$  such that  $d_{n-l}^m = 0$  then the process of elimination can continue if  $d_{n-l}^m = 0$  is replaced by  $\varepsilon > 0$  where  $\varepsilon$  is a very small positive

number. If the condition occurs repeatedly, the turnover process is done using a notation that is different from the previous  $\varepsilon$ .

**Case 2.**

Suppose there is a row with all the elements zero. This indicates the existence of the same pair of magnetized roots, but the roots are different signs or imaginary roots. So it takes an auxiliary polynomial which is formed from the previous row elements of the Hurwitz matrix. This Polynomial Auxiliary will determine the number of roots and root location in the s-plane of a polynomial characteristic equation. In general, the arrangement of Auxiliary Polynomials is formed from even rows on the Hurwitz matrix. Next the zero row elements are replaced by the coefficients of the Polynomial Auxiliary derivative.

**Example 1.**

Given the positive integer  $h$  and  $\varepsilon$  with  $0 < \varepsilon < \frac{1}{h}$ . Determine the stability of the polynomial  $P(z) = hz^5 + hz^4 + 2z^3 + z^2 + \varepsilon z + h$ . Hurwitz matrix can be formed as follows

$$H_5 = \begin{pmatrix} h & 1 & h & 0 & 0 \\ h & 2 & \varepsilon & 0 & 0 \\ 0 & h & 1 & h & 0 \\ 0 & h & 2 & \varepsilon & 0 \\ 0 & 0 & h & 1 & h \end{pmatrix}$$

We can count the new even rows using

$$b'_{2k} = b_{2k} - \frac{d_0}{d_1} b_{2k-1}$$

And we obtain

$$H_5^{(2)} = \begin{pmatrix} h & 1 & h & 0 & 0 \\ 0 & 1 & \varepsilon - h & 0 & 0 \\ 0 & h & 1 & h & 0 \\ 0 & 0 & 1 & \varepsilon - h & 0 \\ 0 & 0 & h & 1 & h \end{pmatrix}$$

Based on the theorem 2 polynomial  $P(z)$  is unstable because of  $d_4^{(2)} = \varepsilon - h < 0$ . But if you only see the main minor positivity of  $H_5$ , the process of elimination can be continued. Count new odd rows using

$$b'_{2k+1} = b_{2k+1} - \frac{d_1^{(2)}}{d_2^{(2)}} b_{2k}$$

We obtain

$$H_5^{(3)} = \begin{pmatrix} h & 1 & h & 0 & 0 \\ 0 & 1 & \varepsilon - h & 0 & 0 \\ 0 & 0 & 1 + h^2 - \varepsilon h & h & 0 \\ 0 & 0 & 1 & \varepsilon - h & 0 \\ 0 & 0 & 0 & 1 + h^2 - \varepsilon h & h \end{pmatrix}$$

Growth factors are obtained

$$\rho = \frac{1 + h^2 - \varepsilon h}{h} > h$$

since polynomial  $P(z)$  unstable resulting in growth factors greater than  $h$ .

**Example 2.**

Determine the stability of the polynomial

$$P(z) = z^6 + 2z^5 + 8z^4 + 12z^3 + 20z^2 + 16z + 16.$$

Polynomial  $P(z)$  can be set as the Hurwitz matrix as follows

$$H_6^{(1)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 1 & 8 & 20 & 16 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 1 & 8 & 20 & 16 & 0 \\ 0 & 0 & 2 & 12 & 16 & 0 \\ 0 & 0 & 1 & 8 & 20 & 16 \end{pmatrix}$$

We count the new even rows using

$$b'_{2k} = b_{2k} - \frac{d_0}{d_1} b_{2k-1}$$

We obtain

$$H_6^{(2)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 0 & 2 & 12 & 16 & 0 \\ 0 & 0 & 2 & 12 & 16 & 0 \\ 0 & 0 & 0 & 2 & 12 & 16 \end{pmatrix}$$

Count the new odd rows using

$$b'_{2k+1} = b_{2k+1} - \frac{d_1^{(2)}}{d_2^{(2)}} b_{2k}$$

and

$$H_6^{(3)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 12 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 12 & 16 \end{pmatrix}$$

Seen in the third and fifth rows where  $d_3^{(3)}, d_5^{(3)} = 0$  or in other words all elements in the third and fifth rows are zero. So that the Auxiliary Polynomial  $P(z)$  is formed from the second and fourth row coefficients,

$$P(z) = 2z^4 + 12z^2 + 16$$

The third and fifth row coefficients are obtained from the following equation,

$$\frac{dP(z)}{dz} = 8z^3 + 24z$$

So we get the matrix as follows

$$H_6^{(3)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 0 & 8 & 24 & 0 & 0 \\ 0 & 0 & 2 & 12 & 16 & 0 \\ 0 & 0 & 0 & 8 & 24 & 0 \\ 0 & 0 & 0 & 2 & 12 & 16 \end{pmatrix}$$

We count the new even rows using

$$b'_{2k} = b_{2k} - \frac{d_0}{d_1} b_{2k-1}$$

We obtain,

$$H_6^{(4)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 0 & 8 & 24 & 0 & 0 \\ 0 & 0 & 0 & 6 & 16 & 0 \\ 0 & 0 & 0 & 8 & 24 & 0 \\ 0 & 0 & 0 & 0 & 6 & 16 \end{pmatrix}$$

we calculate the new odd number by using,

$$b'_{2k+1} = b_{2k+1} - \frac{d_1^{(2)}}{d_2^{(2)}} b_{2k}$$

If

$$H_6^{(5)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 0 & 8 & 24 & 0 & 0 \\ 0 & 0 & 0 & 6 & 16 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 & 16 \end{pmatrix}$$

Count the new even rows using

$$b'_{2k} = b_{2k} - \frac{d_0}{d_1} b_{2k-1}$$

We obtain

$$H_6^{(6)} = \begin{pmatrix} 2 & 12 & 16 & 0 & 0 & 0 \\ 0 & 2 & 12 & 16 & 0 & 0 \\ 0 & 0 & 8 & 24 & 0 & 0 \\ 0 & 0 & 0 & 6 & 16 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 \end{pmatrix}$$

Further simplified

$$H_6^{(6)} = \begin{pmatrix} 1 & 6 & 8 & 0 & 0 & 0 \\ 0 & 1 & 6 & 8 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 8 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that all elements  $d_{2i}^{(k)} > 0$  and  $d_{2i+1}^{(k)} > 0$  for  $i = 1, 2, 3$  so that the auxiliary polynomial roots can be solved as follows,

$$P(z) = 2z^4 + 12z^2 + 16$$

then we got

$$z = \pm 2i \text{ or } z = \pm \sqrt{2}i$$

root of  $P(z)$ .

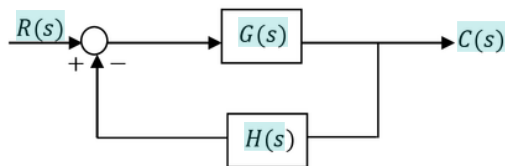
We obtained growth factors  $\rho = \frac{8}{8} = 1$

So that the polynomial characteristic equation based on Routh-Hurwitz conditions is finite stable.

### Example 3

Stability analysis on applications for Routh-Hurwitz conditions

If we had given block diagram of a closed loop system



where  $G(s) = a$  and  $H(s) = \frac{1}{s^3 + 4s^2 + 3s}$   
then the function of the open loop transfer system is

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

and the system output is

$$C(s) = \frac{R(s)G(s)}{1 + G(s)H(s)}$$

So we obtain

$$C(s) = \frac{\frac{a}{s^3 + 4s^2 + 3s}}{1 + \frac{a}{s^3 + 4s^2 + 3s}} = \frac{a}{s^3 + 4s^2 + 3s + a}$$

The characteristic equation of the polynomial is

$$P(s) = s^3 + 4s^2 + 3s + a$$

Hurwitz matrix can be formed as follows

$$H_3 = \begin{pmatrix} 4 & a & 0 \\ 1 & 3 & 0 \\ 0 & 4 & a \end{pmatrix}$$

we count the new even rows using

$$b'_{2k} = b_{2k} - \frac{d_0}{d_1} b_{2k-1}$$

We obtain

$$H_3^{(2)} = \begin{pmatrix} 4 & a & 0 \\ 0 & 3 - \frac{a}{4} & 0 \\ 0 & 4 & a \end{pmatrix}$$

we count the odd number of new ones using

$$b'_{2k+1} = b_{2k+1} - \frac{d_1^{(2)}}{d_2^{(2)}} b_{2k}$$

And we obtain

$$H_3^{(3)} = \begin{pmatrix} 4 & a & 0 \\ 0 & 3 - \frac{a}{4} & 0 \\ 0 & 0 & a \end{pmatrix}$$

Note  $a > 0$  so that it is positive and for  $3 - \frac{a}{4} > 0$  where  $a > 12$ . So we get  $0 < a < 12$  as stable system.

We get the optimal growth factor

$$\rho = \frac{a}{a} = 1.$$

## 5. Conclusions

Suppose that given a polynomial which has every zero of the polynomial is located in the half left open field if and only if  $\mathbf{d}_k > \mathbf{0}$  for all  $k = 0, 1, \dots, n$  and  $\mathbf{detH}_n > \mathbf{0}$ . Every zero generator of the polynomial  $P(z)$  is located in the half left open field if and only if all steps can be taken from the existing elimination procedure. The elimination procedure uses Gauss Elimination with partial pivots. The optimal growth factor from the polynomial characteristic equation for a stable Routh-Hurwitz condition is 1, else not stable.

## 6. Acknowledgment

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# Stability Analysis for Routh-Hurwitz Conditions Using Partial Pivot

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Pena, J.M.. "Characterizations and stable tests for the Routh-Hurwitz conditions and for total positivity", Linear Algebra and Its Applications, 20041201

Publication

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